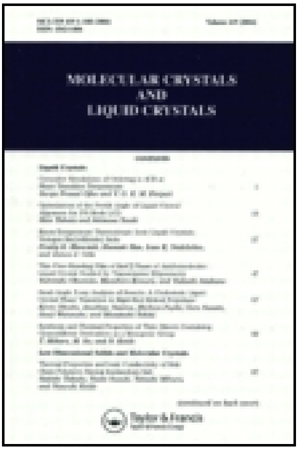

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H. Yamada [a] & C. Ishii [a]
[a] Department of Physics, Science University of Tokyo, Shinjuku, Tokyo, 162, Japan
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Relaxation of Distortions in a Nematic Thin Film with Weak Anchoring Condition†

H. YAMADA and C. ISHII
Department of Physics, Science University of Tokyo, Shinjuku, Tokyo, 162, Japan

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Dynamical properties of a nematic thin film are investigated under the weak anchoring condition on the boundaries. By solving the equations for small distortions, their asymptotic behaviors are found. Numerical solutions of nonlinear equations for the director field are obtained both in case of small and large distortions around the lower and upper threshold fields. The asymptotic behaviors of these solutions are compared with the analytical results. It is concluded that the dynamics around the upper critical field is governed by the boundaries and strongly space dependent.

1 INTRODUCTION

Distortions of director field $\mathbf{n}(\mathbf{r})$ in a thin nematic liquid crystal, induced by the applied electric or magnetic field, has been extensively investigated because of its importance in the application to display devices.

The main physics in this system is the competition between the bulk (elastic and external) torques and the surface anchoring force which tend to align the molecular axis in different directions. It is well known that the system shows a second order Freedericksz transition at a threshold field H' from a uniform undistorted state (set up by the surface force) to a distorted state. In addition, if the anchoring force is weakened, the system shows the second transition at the higher threshold field H'' from the distorted state to another uniform undistorted state in which the molecules are completely aligned in the field direction. The static properties of such a weakly anchored nematic thin film (to be called WANTF) has been fully worked out by Nehring, Kmetz and Scheffer.[1]

† A part of this work has been reported at the Eighth International Liquid Crystal Conference, Kyoto, July 3, 1980.

They stressed the importance of WANTF in improving the multiplexing capability of the display systems.

In this paper, we try to investigate the dynamics of distortions of WANTF, especially the relaxation (or the nucleation and saturation) of distorted state to (from) the undistorted states in $H < H'$ and $H > H''$, induced by a sudden switching of the external field. The present article constitutes a natural extention of the work by Pieranski, Brochard and Guyon[2] who performed the corresponding analysis of small distortions in the rigidly anchored film. Large distortions of rigidly anchored nematic thin films have been extensively investigated in terms of the numerical method by Berreman[3,4] and van Doorn.[5,6]

Since we are primarily interested in the qualitative effect of weak anchoring force, we confine ourselves throughout this paper to the simplest model, characterized by the following assumptions: (1) The differences in the elastic constants $K_{ii}(i = 1, 2, 3)$ are neglected. (2) No hydrodynamical coupling is considered.

We first summarize in §2 the basic equations and approximations. In §3, we analyze the asymptotic behavior of small distortions both around the lower and upper threshold fields H' and H''. The results of the numerical solutions for the large distortions are presented in §4, and interpreted in terms of the results derived in §3. The final section is devoted to the discussion.

2 BASIC EQUATIONS AND APPROXIMATIONS

Let us consider a thin layer of nematic liquid crystal with thickness d, sandwiched between the boundary plates. In the following, we consider the geometry I of Ref. 2 where the director is aligned parallel to the boundary plates in the absence of field, and the field is applied perpendicular to the boundaries. (Another geometry, geometry III of Ref. 2, is treated in a similar fashion with minor modifications.)

For the sake of simplicity, we approximate the Frank elastic constants by a single parameter K and we also leave out the coupling between the director and velocity fields. In this simplified situation, the equation of motion for the director field $\mathbf{n}(z, t) = [\cos\theta(z, t), 0, \sin\theta(z, t)]$ is written down as

$$K \cdot \partial^2\theta/\partial^2 z + \chi_a H^2 \sin\theta \cos\theta = \gamma \cdot \partial\theta/\partial t,$$

where $\chi_a(>0)$ is the susceptibility anisotropy and γ is the rotational viscosity. Here the tilt angle θ is measured from the axis parallel to the boundary, and it depends only on the vertical coordinate z ($z = 0$ is the midpoint between the plates). It is convenient to scale the variables z, t and H by d = thickness, $\tau_0 = \gamma d^2/K$ = rigid film relaxation time and $H_0 = \pi/d(K/\chi_a)^{1/2}$ = rigid film threshold field, respectively; $x \equiv z/d$, $t' = t/\tau_0$, $h = H/H_0$. The lower and upper threshold fields are denoted by h' and h'' in this unit. In the new units, the above equation of motion is written in the form:

$$\partial^2\theta/\partial x^2 + \tfrac{1}{2}(\pi h)^2 \sin 2\theta = \partial\theta/\partial t, \tag{2.1}$$

where we abbreviated the prime on t'.

In order to take account of the effect of anchoring force, we assume the balance between the anchoring torque and elastic torque upon the surface molecules. This yields the equations for the surface tilt angles of the form

$$\frac{\partial\theta(x, t)}{\partial x} \pm \frac{\pi}{2\lambda} \sin 2\theta(x, t) = 0, \quad x = \frac{1}{2} \tag{2.2}$$

where $\lambda = \pi K/Cd$ measures the strength of anchoring force, C being the surface coupling energy introduced by Nehring *et al.*[1]

For later use, we briefly summarize here the set of equations, describing the equilibrium distorted configurations $\theta_0(x)$ of WANTF in the present geometry. This is given by

$$\sin\theta_0(x) = q\,\frac{\mathrm{cn}(\pi hx, q)}{\mathrm{dn}(\pi hx, q)}. \tag{2.3}$$

The parameter $q = q(h, \lambda)$ is determined from

$$\lambda h \cdot \mathrm{dn}(\pi h/2, q) \cdot \mathrm{sn}(\pi h/2, q) = \mathrm{cn}(\pi h/2, q). \tag{2.4}$$

The threshold field h' and h'' are determined from Eq. (2.4) by taking the limit $q \to 0$ or 1,

$$\lambda h' = \cot(\pi h'/2), \quad \lambda h'' = \coth(\pi h''/2). \tag{2.5}$$

Ref. 1 should be referred to for further details of equilibrium properties.

3 LONG TIME BEHAVIOR OF SMALL DISTORTIONS

In this section, we derive the long time behavior of small distortions, induced by the switching field h across the threshold value h' or h''.

When the field h is initially set at $h_i \lesssim h'$ (or $h_i \gtrsim h''$) and switched to the value $h = h' + \Delta h$ (or $h = h'' - \Delta h$), a small distortion in the tilt angle $\theta(x, t)$ [or $\phi(x, t) = \pi/2 - \theta(x, t)$] is nucleated and grows up to the equilibrium configuration (2.3). Such a process will be called the saturation process in the following. When the field is switched in the reversed way; $h_i(\gtrsim h') \to h' - \Delta h$ [or $h_i(\lesssim h'') \to h'' + \Delta h$], the initial equilibrium distortion will be simply relaxed. This will be called the relaxation process. In the following, we derived the characteristic time constants of the above processes at the long time after switching the field.

A Relaxation processes

Let us first consider the relaxation of distortion $\theta(x, t)$, when the field is switched from h_i ($\gtrsim h'$) to h ($<h'$) at the instant $t = 0$. The distortion at the time t long after the switching of field is expected to be small, and its dynamical behavior is described by the linearized versions of Eqs. (2.1) and (2.2)

$$\frac{\partial^2\theta}{\partial x^2} + (\pi h)^2\theta = \frac{\partial\theta}{\partial t}, \tag{3.1}$$

$$\frac{\partial\theta}{\partial x} \pm \frac{\pi}{\lambda}\theta = 0, \quad x = \pm\frac{1}{2}. \tag{3.2}$$

This initial value problem is solved in the form

$$\theta(x, t) = \sum_n a_n\theta_n(x)\exp(-t/\tau_n') \tag{3.3}$$

where the functions $\theta_n(x)$ and time constants τ_n' are the solutions of the eigenvalue equation

$$\left[\frac{\partial^2}{\partial x^2} + (\pi h)^2 + 1/\tau_n'\right]\theta_n(x) = 0 \tag{3.4}$$

with the boundary condition (3.2). With aid of the first of Eq. (2.5), these are found to be

$$\theta_n(x) = \cos(\pi h_n' x),$$
$$1/\tau_n' = \pi^2(h_n'^2 - h^2), \quad (n = 1, 2, \ldots). \tag{3.5}$$

Here h_n' ($h_1' < h_2' < \cdots$) denote a series of roots of the equation

$$\lambda h = \cot(\pi h/2),$$

and the lowest eigenvalue h_1' is nothing but the lower threshold field h'. In Eq. (3.3), the coefficients a_n's should be determined according to the initial condition. In the long time limit, only the longest lived mode $\theta_1(x)\exp(-t/\tau_1')$ will survive, and we obtain the asymptotic relaxation law of $\theta(x, t)$ in the form

$$\theta(x, t) = a'\cos(\pi h' x)\exp(-t/\tau_n'), \tag{3.6}$$

$$1/\tau_r' \equiv 1/\tau_1' = \pi^2(h'^2 - h^2), \tag{3.7}$$

irrespective of the magnitude of the initial perturbation $h - h_i$.

The relaxation law of the distortion $\phi(x, t)$ is derived in an analogous manner. To do this, we consider the initial value problem

$$\frac{\partial^2\phi}{\partial t^2} - (\pi h)^2\phi = \frac{\partial\phi}{\partial t}, \tag{3.1'}$$

$$\frac{\partial \phi}{\partial x} \mp \frac{\pi}{\lambda} \phi = 0, \quad x = \frac{1}{2}, \tag{3.2$'$}$$

whose solution may be constructed in terms of the solutions of the eigenvalue equation

$$\left[\frac{\partial^2}{\partial x^2} - (\pi h)^2 + 1/\tau_n'' \right] \phi_n(x) = 0, \tag{3.4$'$}$$

with boundary condition (3.2)′. The solutions of this eigenvalue problem are obtained in the form

$$\phi_0(x) = \cosh(\pi h'' x), \quad 1/\tau_0'' = \pi^2(h^2 - h''^2), \tag{3.5$'$}$$

and

$$\phi_n(x) = \cos(\pi h_n'' x), \quad 1/\tau_n'' = \pi^2(h^2 + h_n''^2), \quad [n = 1, 2, \ldots]. \tag{3.5$''$}$$

In the above expressions, h'' is the upper threshold field, and h_n'' are a series of the imaginary roots of the equation

$$\lambda h = \coth(\pi h/2).$$

It is interesting to note that there appears a special surface mode $\phi_0(x)$ with maximum amplitude at boundaries, in addition to the modes $\phi_n(x)$ ($n \geqq 1$) with similar space structures to $\theta_n(x)$. It is clear that the long time behavior of the distortion $\phi(x, t)$ is governed by the mode $\phi_0(x)$, and its asymptotic relaxation law takes the form

$$\phi(x, t) = a'' \cosh(\pi h'' x) \exp(-t/\tau_r''), \tag{3.6$'$}$$

$$1/\tau_r'' \equiv 1/\tau_0'' = \pi^2(h^2 - h''^2). \tag{3.7$'$}$$

B Saturation processes

Now let us consider how the distortion $\theta(x, t)$ behaves asymptotically, when the field is switched from h_i ($\lesssim h'$) to $h = h' + \Delta h$ at the instant $t = 0$. At a large t, the distortion is assumed to have the form

$$\theta(x, t) = \theta(x, \infty) - \Delta\theta(x, t), \tag{3.8}$$

where $\theta(x, \infty)$ denotes the equilibrium configuration (2.3) at the field $h = h' + \Delta h$. Since $\Delta\theta$ is expected to be small in the long time limit, we put (3.8) into Eq. (2.1) and linearize it with respect to $\Delta\theta$. In this linearized equation, t plays a role of cyclic variable and $\Delta\theta$ may be written in the form

$$\Delta\theta(x, t) = f(x) \cdot \exp(-t/\tau_s'). \tag{3.9}$$

The function f obeys the equation

$$\left[\frac{d^2}{dx^2} + (\pi h)^2 \cos 2\theta(x, \infty) + 1/\tau_s'\right] f(x) = 0 \qquad (3.10)$$

which should be solved as an eigenvalue problem with boundary condition

$$\left[\frac{d}{dx} \pm \frac{\pi}{\lambda} \cos 2\theta(x, \infty)\right] f(x) = 0, \quad x = \frac{1}{2}. \qquad (3.11)$$

We solve this eigenvalue problem by the ordinary perturbation method, assuming the parameter $\Delta h/h'$ to be small. Thus the function $f(x)$ and constant $1/\tau_s'$ are expanded in the way

$$f(x) = f^{(0)}(x) + f^{(1)}(x) + \ldots, \quad 1/\tau_s' = 1/\tau^{(0)} + 1/\tau^{(1)} + \ldots.$$

It is easy to see that the lowest order function $f^{(0)}$ satisfies

$$\left[\frac{d^2}{dx^2} + (\pi h')^2 + 1/\tau^{(0)}\right] f^{(0)}(x) = 0, \qquad (3.12)$$

and the boundary condition

$$\left[\frac{d}{dx} \pm \frac{\pi}{\lambda}\right] f^{(0)}(x) = 0, \quad x = \frac{1}{2}. \qquad (3.13)$$

This eigenvalue problem is identical with that considered in Eqs. (3.4) and (3.2), except that the field h there is replaced by h'. This means that the lowest eigenmode $\theta_1(x)$ has infinite life time $1/\tau_1' = 0$. In other words, the lowest order contribution to the inverse saturation time vanishes, and the space structure of the distortion $\Delta\theta(x, t)$ will tend to the form

$$f^{(0)}(x) = b' \cos(\pi h' x), \qquad (3.14)$$

where the constant b' should be determined from the initial condition.

We now proceed to the analysis to the next order solution $f^{(1)}(x)$ which obeys the equation

$$\left[\frac{d^2}{dx^2} + (\pi h')^2\right] f^{(1)}(x)$$
$$= (\pi h')^2 f^{(0)}(x) \{2\theta^2(x, \infty) - 2\Delta h/h' - 1/[(\pi h')^2 \tau^{(1)}]\}, \qquad (3.15)$$

with the boundary condition

$$\left[\frac{d}{dx} \pm \frac{\pi}{\lambda}\right] f^{(1)}(x) = \pm \frac{2\pi}{\lambda} \theta^2(x, \infty) f^{(0)}(x), \quad x = \pm \frac{1}{2}. \qquad (3.16)$$

To solve the Eq. (3.15), we first note that the equilibrium configuration $\theta(x, \infty)$ in the inhomogeneous term is expressed in the form

$$\theta(x, \infty) = \theta_m \cos(\pi h' x), \quad \theta_m^2 = 8 \frac{\pi h' + \sin(\pi h')}{2\pi h' - \sin(2\pi h')} \frac{\Delta h}{h'}, \tag{3.17}$$

in the present approximation. Then, after some manipulation, we find the solution $f^{(1)}(x)$ of the form

$$f^{(1)}(x) = b' \left\{ -\frac{\theta_m^2}{16} \cos(3\xi) + \left[\frac{3}{4} \theta_m^2 - \frac{\Delta h}{h'} - \frac{1}{2(\pi h')^2} \frac{1}{\tau^{(1)}} \right] \xi \sin \xi \right\}, \tag{3.18}$$

where the constant b' is defined in Eq. (3.14) and $\xi \equiv \pi h' x$. The requirement that the above solution satisfies the boundary condition (3.16) leads to the expression of the time constant $1/\tau^{(1)}$

$$1/\tau^{(1)} = 4(\pi h')^2 \frac{\Delta h}{h'}. \tag{3.19}$$

Summarizing, the long time behavior of the distortion $\theta(x, t)$ in the saturation process around the threshold field h' is described by the formula

$$\theta(x, t) = \theta(x, \infty) - b' \cos(\pi h' x) \cdot \exp(-t/\tau_s'), \tag{3.20}$$

where the lowest order expression of the inverse saturation time $1/\tau_s'$ is given in Eq. (3.19).

We now touch upon the saturation law for the distortion $\phi(x, t)$, induced by switching field from h_i ($\gtrsim h''$) to $h'' - \Delta h$. This is derived in an analogous manner as that for $\theta(x, t)$. The result is summarized in the form

$$\phi(x, t) = \phi(x, \infty) - b'' \cosh(\pi h'' x) \exp(-t/\tau_s''), \tag{3.21}$$

where the inverse saturation time $1/\tau_s''$ is related to the perturbation parameter $\Delta h/h''$ by

$$1/\tau_s'' = 4\,(\pi h'')^2\, \Delta h/h''. \tag{3.22}$$

4 LARGE DISTORTION—NUMERICAL APPROACH

In order to extend the analysis of the previous section to the large amplitude distortions, we have performed the numerical solution of the initial value problem, defined by Eqs. (2.1) and (2.2). This was done by usual procedure of reducing this set of equations to the corresponding difference equations.

However, a special care was needed to obtain the accurate value of $\partial\theta/\partial x$ at the boundaries, especially when we were working around the upper threshold field h''. The special subroutine of Lagrangian method was used for this calculation.

The calculation was performed for the transitions between the distorted and undistorted states, characterized by the field $h' + \Delta h$ (or $h'' - \Delta h$) and $h' - \Delta h$ (or $h'' + \Delta h$); four sets of threshold fields $(h', h'') = (0.94, 10), (0.92, 8), (0.90, 6)$ and $(0.86, 4)$ are considered, corresponding to the anchoring strength $= 0.1, 0.12, 0.17, 0.25$ respectively. For each given h' and h'', following values of Δh were considered; $\Delta h/h'$ (or $\Delta h/h''$) $= 1/4, 1/8, 1/16$, and $1/32$. Thus we obtained the numerical solutions of Eqs. (2.1) and (2.2) in $2 \times 4 \times 4 = 32$ different sets of parameters each for the relaxation and growing processes.

4.1 Switching field across h'

In Figure 1a, a typical example of relaxation profile of the tilt angle is shown as a function of x at various times $t = nt'$, $t' = 0.48$ in the unit of $\tau_0 = \gamma d^2/K$, after switching the field from $h' + \Delta h$ to $h = h' - \Delta h$ ($h' = 0.864$, $\Delta h = h'/16 = 0.054$). Plotted in Figure 2 is the relaxation of θ at $x = 0$ (center of film) and $x = \pm\frac{1}{2}$ (boundaries).

Expecting the behavior of the form Eq. (3.6), we put this result in another way in Figure 3 where $\ln [\theta(x, 0)/\theta(x, t)]$, ($x = 0, \pm\frac{1}{2}$) is plotted against t. Reading the asymptotic slopes of these curves, we extract the values of $1/\tau_r'$. These are summarized in Figure 4, showing a good agreement with the prediction of Eq. (3.7).

We show in Figure 1b how the small distortion grows up when the field is switched just in the opposite direction as the above case; $h' - \Delta h \rightarrow h = h' + \Delta h$. The Figures 5 and 6 are drawn in order to read the saturation time $1/\tau_s'$. As the initial configuration, we took the function of the form $\theta(x, 0) = \epsilon\cos(\pi h' x)$, ϵ being chosen to be 10^{-2} times the equilibrium maximum tilt angle. The saturation times thus obtained are plotted in Figure 7 together with the prediction of Eq. (3.10), showing again a reasonable agreement.

4.2 Switching field across h''

The Figures 8a, 9 and 10 show the results for the relaxation of distortion from the equilibrium distorted state at the field $h'' - \Delta h$ to purely homeotropic state at $h'' + \Delta h = h$ ($h'' = 4.0$, $\Delta h = h''/16 = 0.25$). When Figures 9 and 10 are compared with Figures 2 and 3, it is seen that the initial behavior of ϕ at the boundary points is appreciably different from that at the center; the distortion diminishes faster at the center than the boundaries. However the distortion ϕ

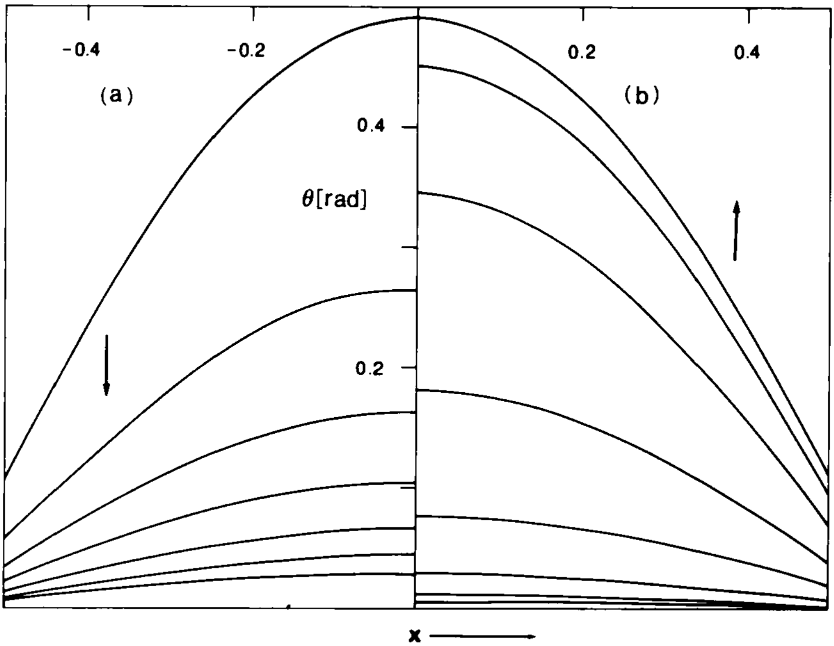

FIGURE 1 (a) Tilt angle θ at various times $t = nt'$; $n = 0,1,2,3,4,5,6$ from top to bottom curves, after switching field from $h' + \Delta h$ to $h' - \Delta h = h$. (b) Tilt angle θ at various times $t = nt'$; $n = 0,2,4,6,8,10$ and 12 from bottom to the second top curves, after switching the field from $h' - \Delta h$ to $h' + \Delta h = h$. The top curve shows the equilibrium distortion $\theta_0(x, h)$ at which $\theta(x, t)$ arrives at $t \approx 20t'$. Both in the cases (a) and (b), $h' = 0.864$, $\Delta h = 0.054$ and $t' = 0.48$ in unit τ_0.

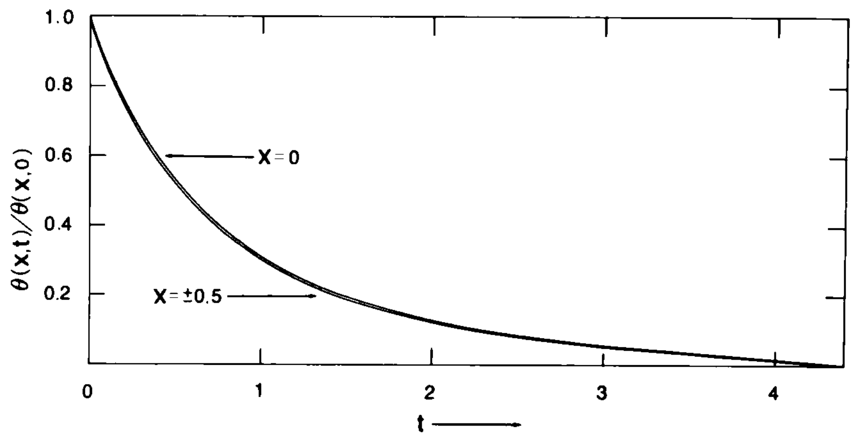

FIGURE 2 Relaxation of distortion θ at the center of film ($x = 0$) and boundaries ($x = \pm\frac{1}{2}$). θ relaxes faster at the boundaries. $h' = 0.864$, $\Delta h = 0.054$.

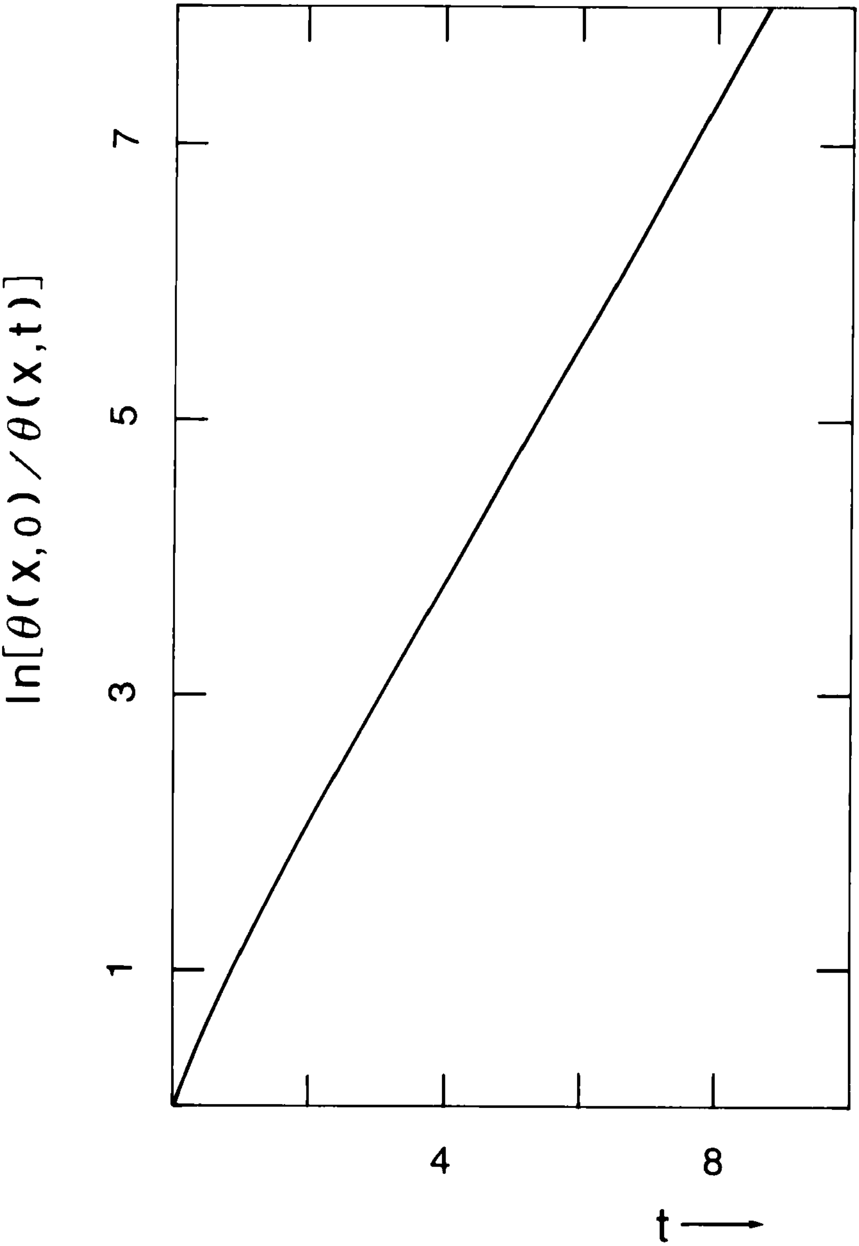

FIGURE 3 Semilogarlithmic plot of the result in Figure 2 to obtain the relaxation time $1/\tau_r'$.

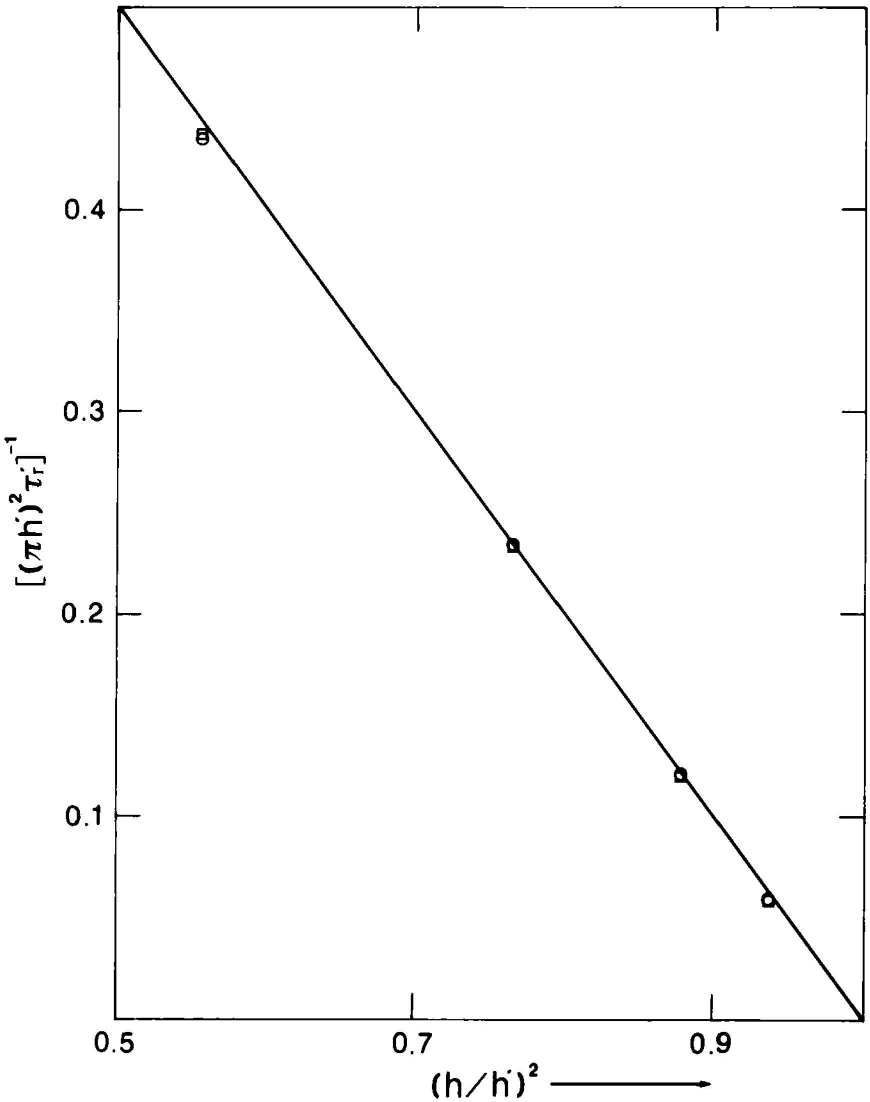

FIGURE 4 Dependence of relaxation time $1/\tau_r'$ on the final state field h and anchoring strength λ. The symbol □ (○) stands for the values obtained numerically for $h' = 0.94$, $\lambda = 0.1$ ($h' = 0.865$, $\lambda = 0.25$). Solid line is the prediction of Eq. (3.7).

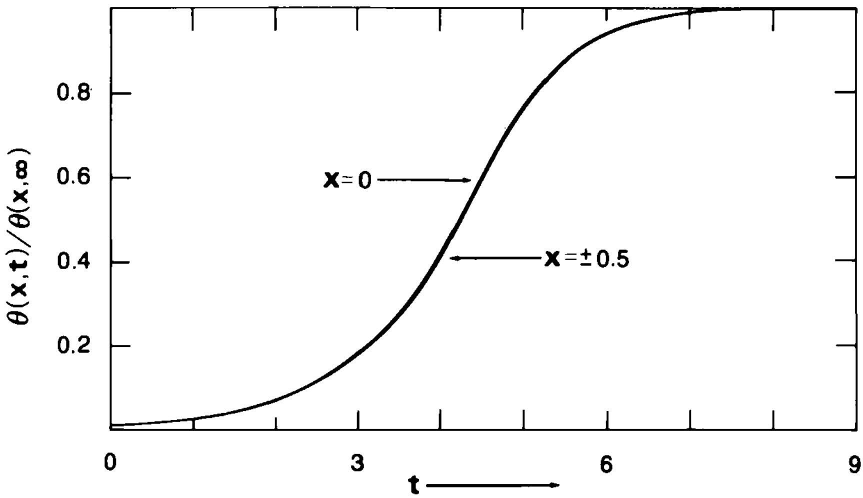

FIGURE 5 Growing and saturation of distortion θ at $x = 0$ (center) and $x = \pm\frac{1}{2}$ (boundaries), corresponding to Figure 1b.

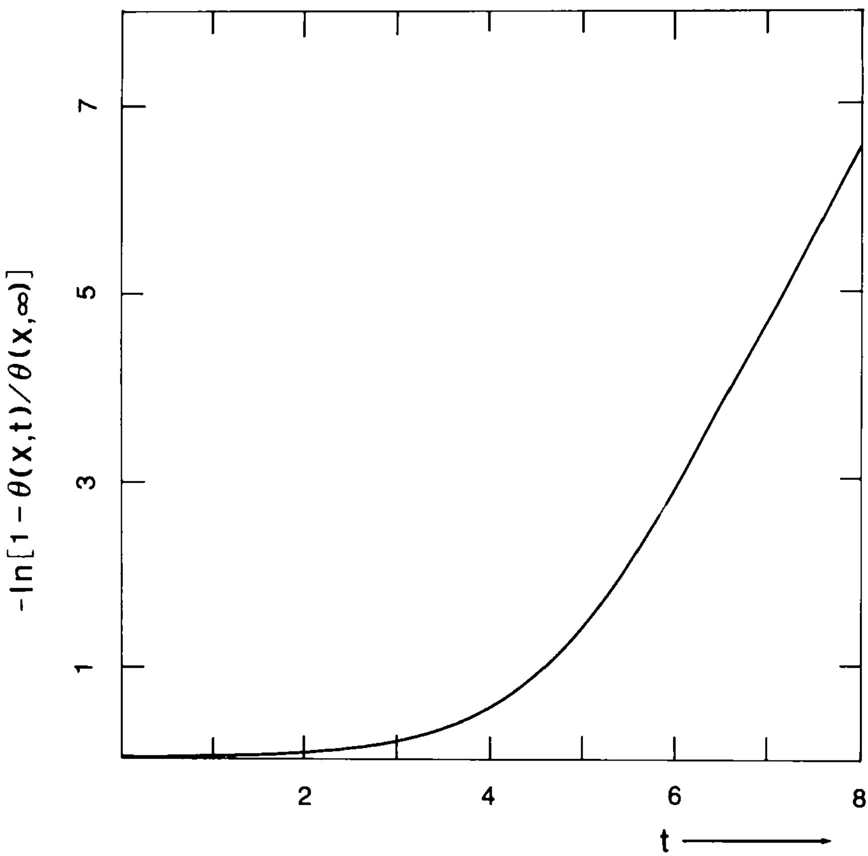

FIGURE 6 Semilogarlithmic plot of $1/[1 - \theta(x, t)/\theta(x, \infty)]$.

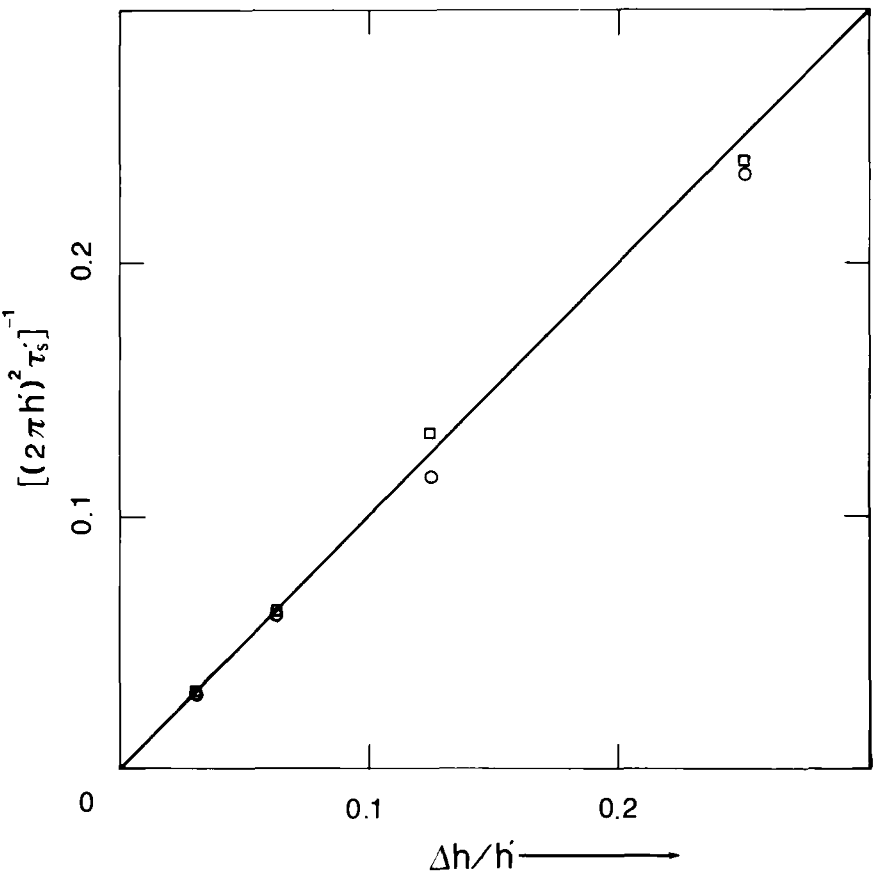

FIGURE 7 Dependence of the saturation time on the final state field h and the anchoring strength λ. The symbol □ (○) stands for the values, numerically obtained for $h = 0.94$; $\lambda = 0.1$ ($h' = 0.865$; $\lambda = 0.25$). Solid line is drawn according to Eq. (3.19).

shows an exponential behavior in the long time both at these points. Thus we plot in Figure 11 the relaxation time $1/\tau_r''$, extracted from the asymptotic slopes of these curves in Figure 10. The results are compared with the prediction of Eq. (3.7)', showing an excellent agreement.

In Figure 8b, we show the saturation profiles of distortion $\phi(x, t)$ from the homeotropic state to the state with field $h = h'' - \Delta h$ ($h'' = 4$, $\Delta h = 0.25$). We see again a remarkable difference in the initial behavior of ϕ at the center and the boundaries. This time, ϕ grows very much faster at the boundaries than the center. However, as seen in Figure 13, the asymptotic behavior of ϕ is again exponential at both points, and we plot the saturation time $1/\tau_s''$ in Figure 14, extracted from the slopes of these curves. The results are compared

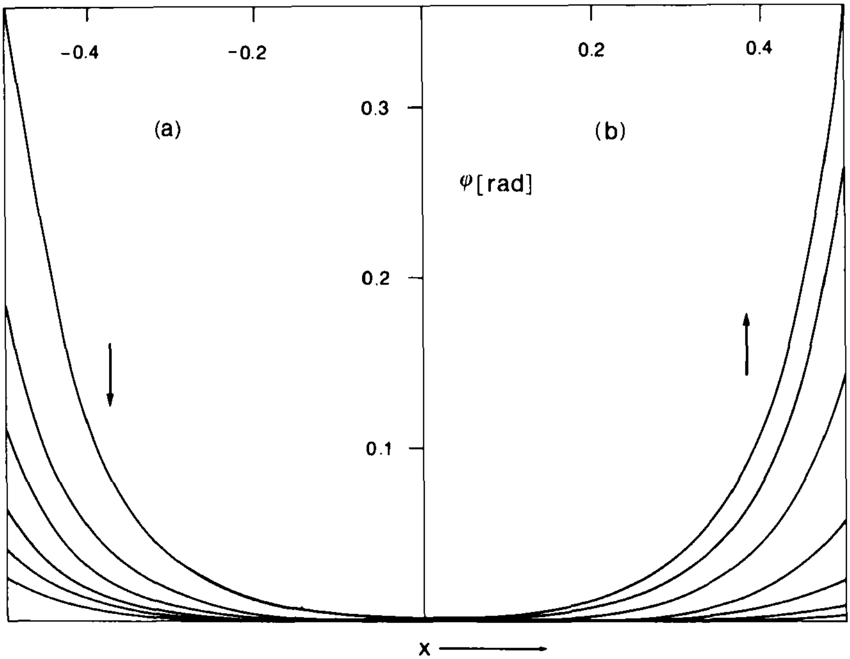

FIGURE 8 (a) Tilt angle ϕ at various times $t = nt''$; $n = 0,1,2,3,4$ and 5 from top to bottom curves, after switching field from $h'' - \Delta h$ to $h'' + \Delta h$. (b) Tilt angle ϕ at various times $t = nt''$; $n = 0,2,4,6,8$ and 10 from bottom to the second top curves, after switching the field from $h'' + \Delta h$ to $h'' - \Delta h$. The top curve represents the ϕ at $t = 18t''$ and almost identical with the equilibrium distortion $\phi_0\ (x,\ h'' - h)$. Both in the cases (a) and (b), $h'' = 4$, $\Delta h = 0.25$ and $t'' = 0.024$.

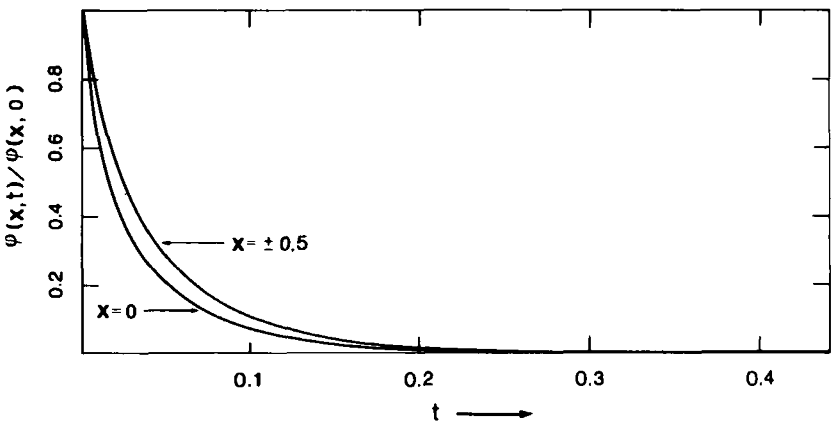

FIGURE 9 Relaxation of distortion ϕ is faster at $x = 0$ (center) than at $x = \pm\frac{1}{2}$ (boundaries); $h'' = 4$, $h = 0.25$.

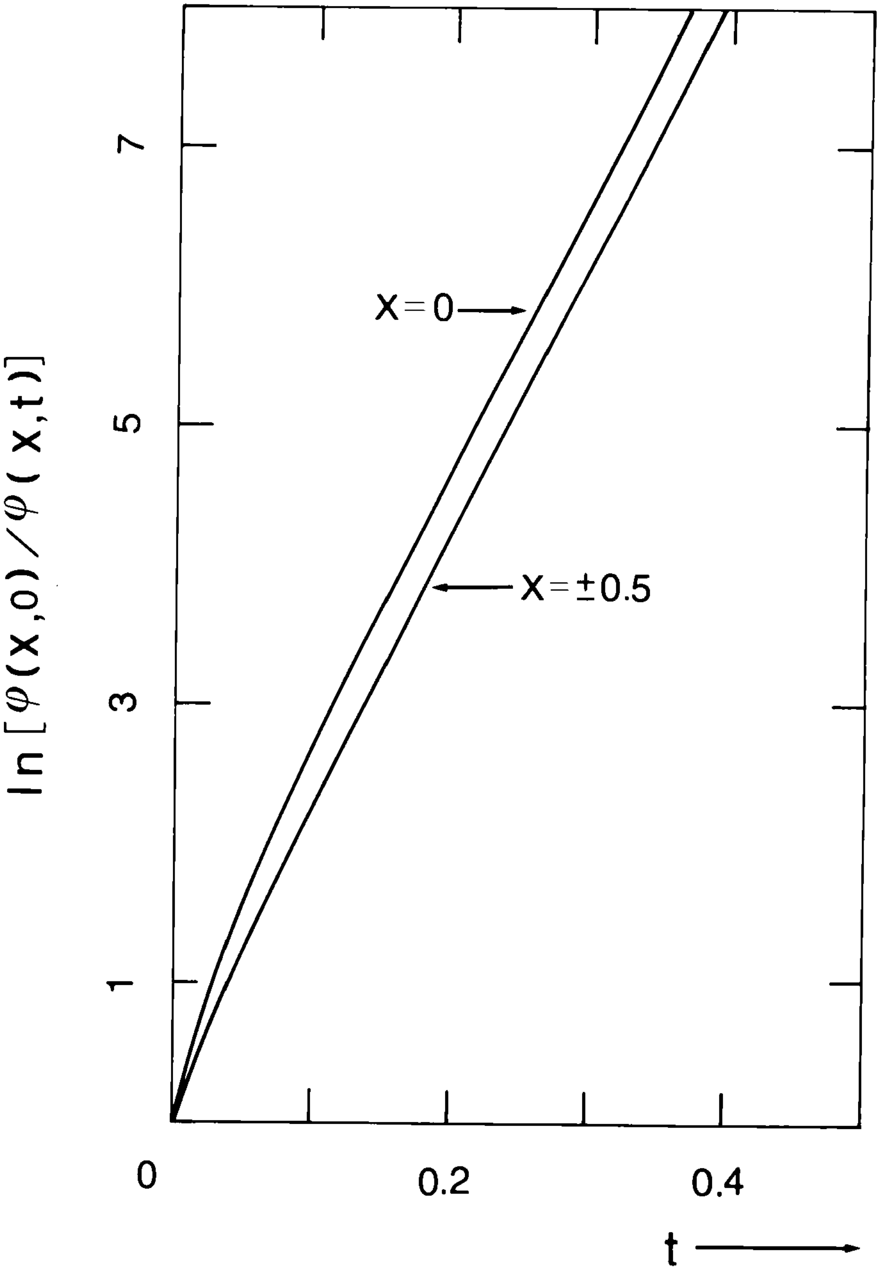

FIGURE 10 Semilogarlithmic plot of ϕ $(x, 0)/\phi$ (x, t) to extract $1/\tau''_r$.

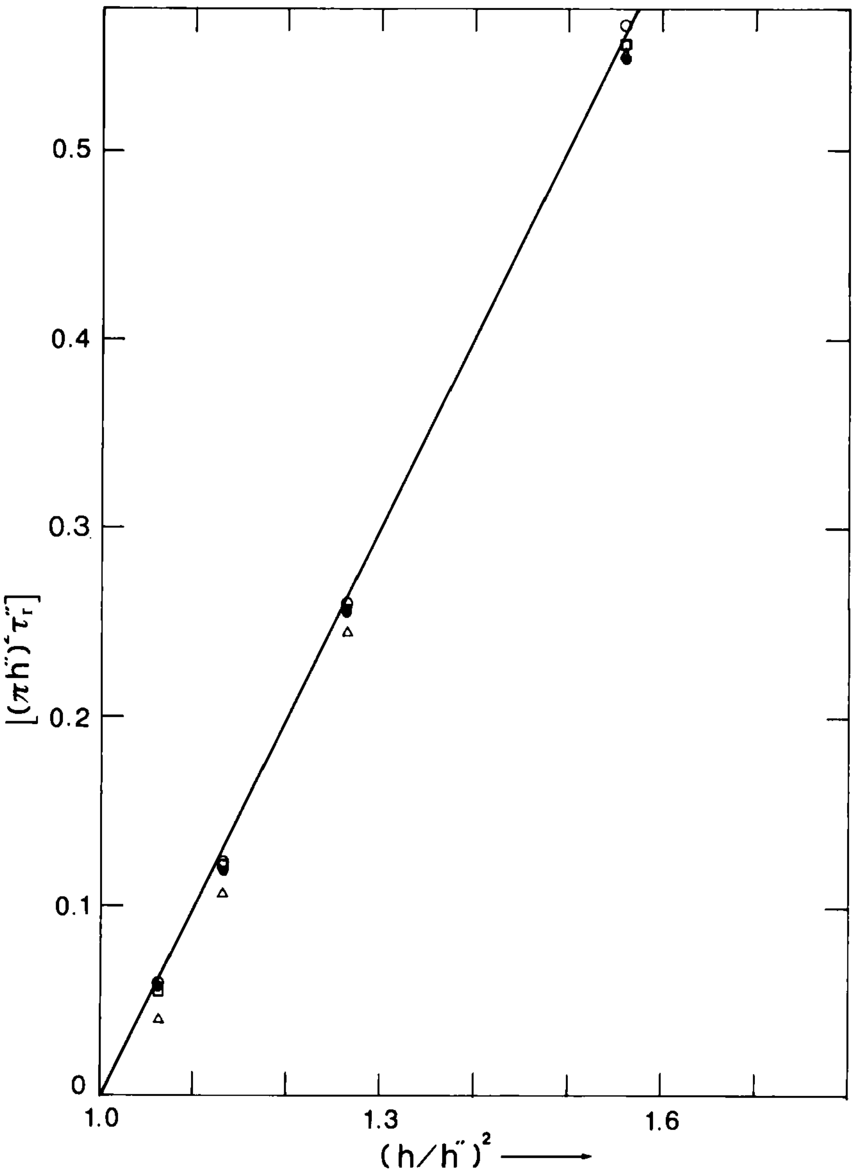

FIGURE 11 Dependence of $1/\tau_r''$ on the final state field and anchoring strength λ. The symbols □, △, ●, ○ correspond to (h'', λ) = (10, 0.1), (8, 0.12), (6, 0.17) and (4, 0.25) respectively. Solid line is the result of Eq. (3.7)′.

with the theoretical prediction (3.22). The agreement is limited only in the very small perturbation $\Delta h/h''$.

As discussed above, the distortion $\phi(x, t)$ behaves appreciably differently at the center and the boundaries, and this is interpreted as a natural consequence of the specific physical roles of the surface molecules in the phase transition at the upper threshold field h''. In the relaxation to the homeotropic structure, the increase in the field beyond the value h'' is first felt by the bulk molecules, and it is only through the propagation of this order in the bulk that surface molecules align themselves along the field, overcoming the influence of the surface force. This means that the distortion ϕ relaxes faster at the center than boundaries, as is just seen in Figure 9. On the other hand, in the saturation process, the decrease in the field down to the value less than h'' is first responded to by the surface molecules. Distortion is thus first nucleated in the surface region and then propagated into the bulk again through the elastic coupling. This results in the faster growing of distortion at the boundaries than the center, as is seen in Figure 12.

5 CONCLUSION

We have studied the dynamics of distortions in the director field of the weakly anchored nematic thin film (WANTF) both in terms of analytical and numerical approaches, with special attention to what happens in the vicinity of upper threshold field h''.

As far as the relaxation processes are concerned, it is shown that their asymptotic behavior is clearly understood in terms of the relaxation eigen-

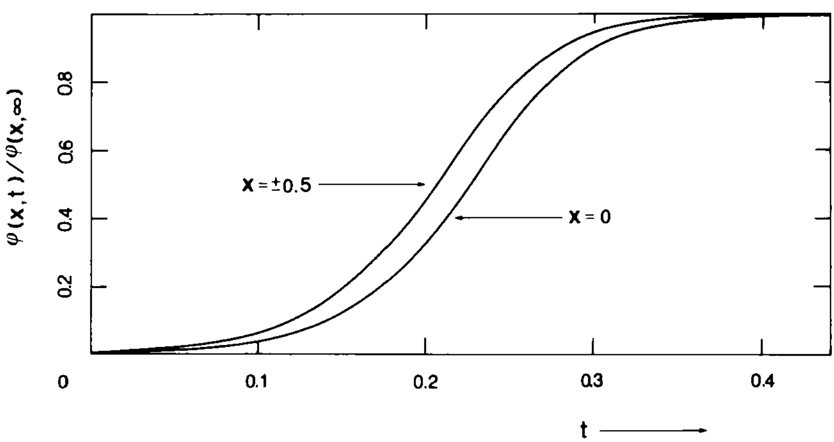

FIGURE 12 Growing distortion ϕ. The distortion ϕ grows considerably faster at the boundaries than at the center.

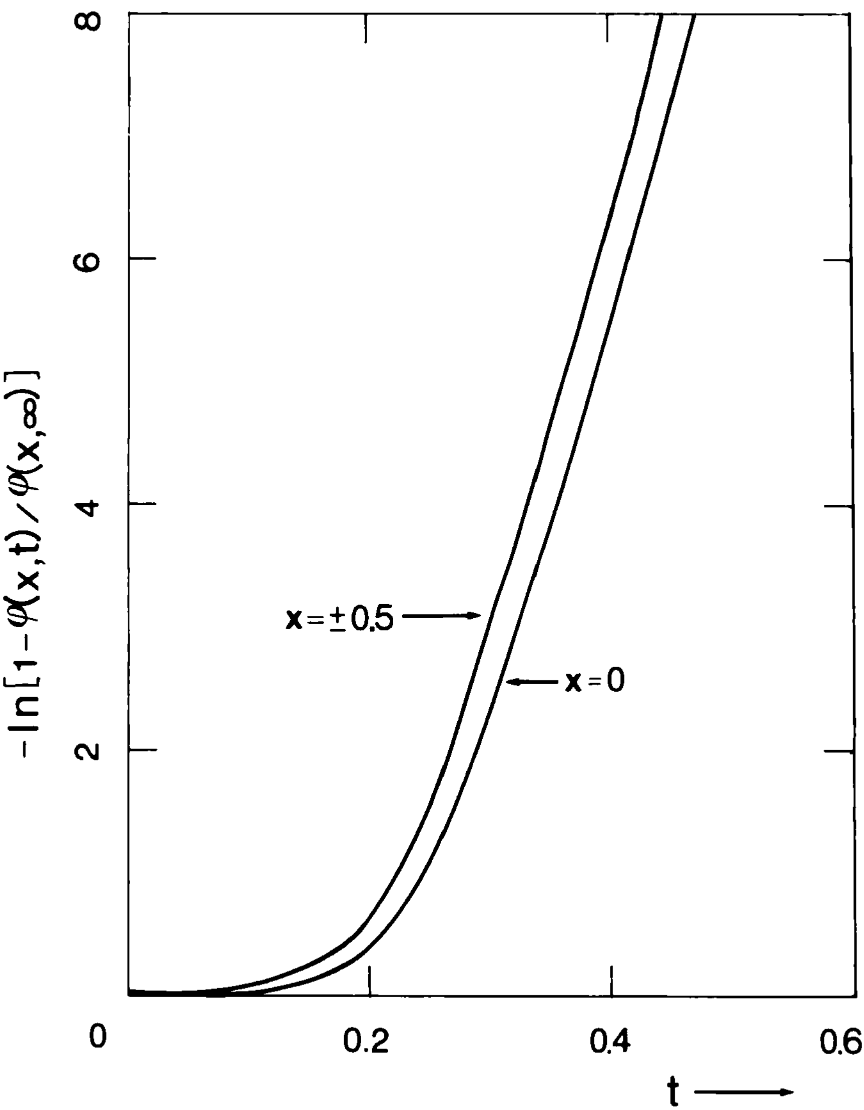

FIGURE 13 Semilogarlithmic plot of $1/[1 - \phi(x, t)/\phi(x, \infty)]$.

modes, $\theta_n(x)$ or $\phi_n(x)$. In the relaxation process across the lower threshold field h', the mode $\theta_1(x)$ plays a dominant role and its relaxation time determines the asymptotic behavior of the distortion $\theta(x, t)$. On the other hand, in the relaxation to the high field undistorted state across h'', the surface mode $\phi_0(x)$ governs the asymptotic behavior of distortion $\phi(x, t)$.

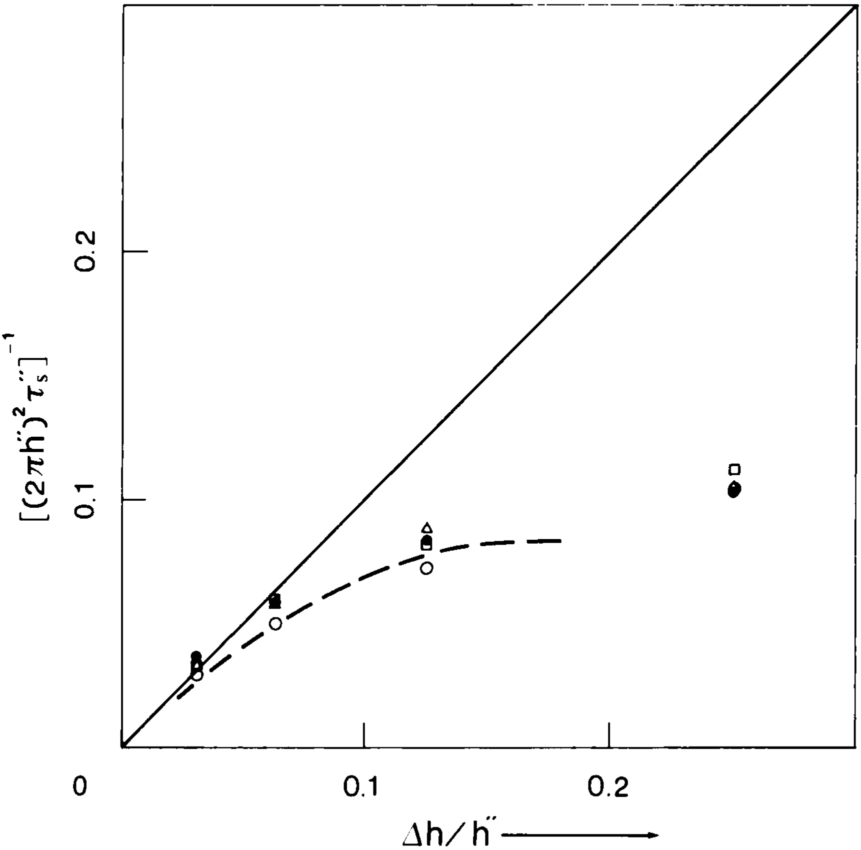

FIGURE 14 Dependence of $1/\tau_s''$ on the final state field and anchoring strength. The symbols □, △, ● and ○ correspond to $(h'', \lambda) = (10, 0.1), (8, 0.12), (6, 0.17)$ and $(4, 0.25)$, respectively. The solid line shows the result of Eq. (3.22), and the dotted curve represents $1/\tau_s''^{(1)} + 1/\tau_s''^{(2)}$.

The situation is a bit complicated in the saturation processes. Here, the analytical approach is possible only in case of small magnitude of switched field Δh. Approximate expressions of the time constants, describing the asymptotic behavior of distortions, are derived by perturbation theory. As far as the saturation process across the field h' is concerned, the formula thus obtained works well to interpret the results of numerical experiment. On the other hand, in regards to the saturation processes across the upper threshold field h'', there appears a considerable discrepancy even at a modest value of perturbation parameter $\Delta h/h''$. In order to improve the agreement, we tried to calculate the higher order correction to the inverse saturation time $1/\tau_s''$. The result is quite complicated in case of general anchoring strength λ. Therefore we write here the expression, valid in the strong anchoring limit,

$$1/\tau_s''^{(2)} = -12(\pi h'')^2 \left(\frac{\Delta h}{h''}\right)^2 .$$

This correction certainly improves the agreement, as is seen in Figure 14.

The single elastic constant approximation $K_{11} = K_{22} = K_{33}$ has been employed throughout the paper. We have performed an analogous analysis as given in §3 also for the case where $\kappa = K_{11} - K_{33} \neq 0$. The static properties are significantly affected by the presence of nonzero κ, but it is confirmed that the expressions of various time constants such as Eqs. (3.7), (3.7)′, (3.19) and (3.22) are all held in this general case.

In conclusion, it should be emphasized that dynamical behavior of WANTF in the vicinity of the upper threshold field h'' is characterized by a strong space dependence, reflecting the specific physical roles of the surface molecules in the phase transition at this field.

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